On Laws of Work Organization in Human Cooperation

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ABSTRACT

From the perspective of cognitive informatics (CI), this paper proposes internal relations between distance and orientation knowledge of extended objects, and presents a formal representation of spatial knowledge. The connection *relation is taken as primitive. Notions of near extension regions and the* nearer *predicate are developed. Distance relations between extended objects are understood as degrees of the* near *extension from one object to the other. Orientation relations are understood as distance comparison from one object to the sides of the other object. Therefore, distance and orientation relations can be internally related through the* connection *relation. The notion of the* fiat projection *is presented to model the mental formation of the deictic orientation reference framework. This article introduces a new axiom to govern the* connection *relation in the literature and presents examples to show diagrammatically the internal relations between distance and orientation relations of extended objects.*

Keywords: cognitive informatics; decision optimization; human cooperative work; interchangeability of labor and time; management science; optimal labor allocation; organization laws; shortest duration determination; software engineering; system science; Wang's laws

INTRODUCTION

Human cooperation in groups and projects is a widely generic phenomenon studied in engineering organization in management science, system science, and software engineering (Brooks, 1975; Klir, 1992; Ritzer, 2000). *Cooperative work* is needed when individuals cannot carry out a given work or solve a certain problem. Brooks presented an empirical study on the myths of the relationship between labor (number of persons) and time (duration) in the software engineering context (Brooks, 1975), where the author left the conclusion open on "propositions of the mythical man-month: true or false?"

From the perspective of cognitive informatics (CI), i.e. Wang (2003), we are particular interested in internal relations among different kinds of spatial knowledge. We know that distance knowledge and orientation knowledge can be acquired through perception, and however they denote different semantics in spatial linguistic descriptions. The aim of the article is to present internal relations between them.

In a broad perspective, a theory of cooperative work organization is at the center of management and system sciences. The development of management as a scientific discipline can be traced back to the work of Frederick Taylor in the 1890s on improvement of operations in production. Taylor's work on *Principles of scientific management* inaugurates management as a formal branch of human inquiry and knowledge on cooperative work and industrial engineering (Tayler, 1911). Henry Gantt studied project scheduling and developed the control chart in the 1900s known as Gantt Chart (Gantt, 1919) for minimizing interrelated job completion times. In the 1920s, William Shewhart introduced statistics into management and developed the control charts for statistical process and quality control (Shewhart, 1939). In the 1950s, project scheduling was intensively studied and the program evaluation and review technique (PERT) (Dougherty & Stephens, 1984; Hagstrom, 1988; Schonberger, 1981) and critical path method (CPM) (Kelley, 1961; Schonberger, 1981) were developed. Various programming methods were proposed to solve optimization problems for a given objective and a number of constraints such as linear programming in the 1940s (Murty, 1983), nonlinear programming and dynamic programming in the 1950s and later (Bertsekas, 1995; Donnelly, Gibson, & Ivancevich, 1998; Schmenner & Swink, 1998).

The classic management thoughts are developed in Henri Fayol's work on *General and industrial management* in 1929 (Fayol, 1929) and James Mooney's work on *The principle of organization* in 1947 (Mooney, 1947). Fayol was interested in the basic principles of management on determining "soundness and good working order (Fayol, 1929)." Mooney views organization as the technique of relating specific duties or functions in a coordinated whole.

Organization is a management process that coordinates and allocates essential means such as labor, resources, and processes in order to implement a planned work. A *group* is a basic unit in organization where multiple persons working together by cooperation toward a common goal in production or service (Ritzer, 2000; Schonberger, 1981). Groups are needed because the *interdependency* among members when a given work cannot be carried out by individuals limited by either *resource dependency* or *functional dependency*. The importance of studies on groups is well explained by Kurt Lewin five decades ago where he wrote: "Although the scientific investigations of group work are but a few years old, I don't hesitate to predict that group work--that is, the handling of human beings not as isolated individuals, but in the social setting of groups--will soon be one of the most important theoretical and practical fields (Zander, 1979)."

Despite a whole spectrum of empirical studies on the age-long problems, there is a lack of a rigorous theory on work organization in human cooperative environment. This article creates a mathematical model for analyzing the mechanisms, behaviors, and laws of cooperative work organization. The basic properties and characteristics of cooperative work and their mathematical models are explored, which explains the transformability between labor and time in cooperative work and the role of the overhead for interpersonal coordination. Then, laws of engineering workloads are derived as a foundation for analyzing the work duration and effort in cooperative project organization. On the basis of the cooperation theories, a set of decision optimization strategies are presented toward optimal project organization for the best labor allocation, the shortest project duration, and the lowest effort. The exchangeability and its constraints between labor and time in engineering project organization are formally analyzed.

FUNDAMENTAL PROPERTIES OF COOPERATIVE WORK IN

ENGINEERING

This section formally analyzes the age-long myth on cooperative work and effort (Brooks, 1975; Fayol, 1929; Mooney, 1947; Tayler, 1911). Mathematical models that explain the equivalence and transformability between labor and time in abstract work organization are systematically developed.

The Properties of Cooperative Workload and Effort

Definition 1. The *workload W* of a cooperative project is determined by a product of the number of labor *L* and the duration *T* needed or spent in the project, for example:

$$W = L \bullet T \quad [PM] \qquad (1)$$

where the unit of labor is *person* (P), the unit of duration is *month* (M), and as a result, the unit of workload is *person-month* [PM].

Although, the concept of workload seems intuitive, there are numerous myths on the relationship between labor and duration as defined in Eq.1 in empirical studies. Major questions remain on the nature of the hybrid product of workload in PM. For example, how many persons and how many months are needed for a given workload? Is 1P • 10M = 10P • 1M = 10PM?

All the practical questions in applications on the nature of cooperative workload can be reduced to the following fundamental problems:

- Whether labor *L* or duration *T* is arbitrarily determinable for a given workload *W* in a cooperative work?
- Are labor *L* and duration *T* interchangeable for a given workload *W* in cooperative work organization?
- If so, what are the constraints and conditions underpinning the interchangeability in cooperative work?

Rigorous investigations into the previous questions will be carried out and formal explanations will be provided throughout the article in the form of a number of mathematical models, theorems, and laws for optimal cooperative work organization.

It is observed that many factors may affect the workload of a cooperative project (Gray, 1989; Hardy & Phillips, 1998; Huseman & Miles, 1988; Huxham, 1996; Okada, Hoshi, & Inoue, 2005; Pasquero, 1991; Roberts & Bradley, 1991; Wang, 2007; Wood & Gray, 1991), such as documentation, swap between roles in a project, and interaction to other groups in an organization. However, a macro indicator known as the *interpersonal coordination rate r* is a unique factor that distinguishes a single person project and a multiple-personal cooperative project. Therefore, the role of *r* is the key to solve the myths on cooperative work organization.

Definition 2. *Interpersonal coordination activities* are tasks that can not be done by an individual, such as communication, meeting, task synchronization, peer review of work products, standardization, supervision, and quality assurance.

The effort on interpersonal coordination activities as a necessary overhead of a cooperative project can be collectively analyzed by the extra time spent by individuals in the project.

Definition 3. The *interpersonal coordination rate r* is an average ratio of the time spent on interpersonal coordination t_r and the total working time of a person *T* in a given project, for example:

$$r = \frac{t_r}{T} \qquad (2)$$

The average rate of interpersonal coordination *r* has a scope of 0 (0%) through 1.0 (100%), where *r* = 0 means there is no interpersonal cooperation and *r* = 100% means all time has been spent on interpersonal cooperation. These are the two extremes that constrain a cooperative work.

For instance, in software engineering, a wide variety of factors may affect the interpersonal coordination rate. Ten major factors such as the scope of project, importance, difficulty, complexity, domain knowledge requirement, experience requirement, special process needed, schedule constraints, budget constraints, and other process constraints have been identified in Wang & King (2000) as summarized in Table 1 where a set of sample weights for the coordination factors is also given.

When the weight for each coordination factor is determined on a measurement scale of 1 through 10, the *average* interpersonal coordination rate needed for the given project can be empirically determined that is proportional to the mathematical mean of the weights as follows:

Table 1. Key factors affecting the rate of interpersonal coordination in software engineering

No.	Factors of project	Scope of Weight (w_i)		
		High	Medium	Low
1	Scope	√		
2	Importance		√	
3	Difficulty	√		
4	Complexity	√		
5	Domain knowledge requirement			√
6	Experience requirement		√	
7	Special process needed		√	
8	Schedule constraints	√		
9	Budget constraints			√
10	Other process constraints		√	

Note: High = 10, Medium = 5, and Low = 1

$$r = \frac{t_r}{T} \propto \frac{\sum_{i=1}^{10} w_i}{100} \qquad (3)$$

Eq. 3 indicates that the empirical range of r in software engineering is $0.1 \le r < 1.0$, or r is between 10% to 99.99%. For example, with the particular layout of a project as given in Table 1, the average interpersonal cooperation rate r is proportional to:

$$r = \frac{\sum_{i=1}^{10} w_i}{100} = (10 + 5 + 10 + 10 + 1 + 5 + 5 + 10 + 1 + 5)/100 = 0.62$$

Surveys on the average interpersonalcooperation rate r in software industry reveal it is in the range from 12.5% to 47.8% (Wang, 2007). The data demonstrated that r may vary in different processes of software engineering, ranging averagely from 49.3% in the design process, 30.8% in the coding process, 60.0% in the integration/testing process, and 47.8% in the maintenance process, respectively. The surveys also shown that different development methodologies may affect the calibration of r. For instance, r =30.2% for projects organized according to conventional waterfall models, and it may be up to50.2% when projects are organized by extreme programming or agile processes.

Definition 4. The *number of interpersonal coordination, n*, needed in a group of size L, $L \ge 2$, is determined by the number of pairwise cooperation in the group, for example:

$$n = C_L^2 = \frac{L!}{2!\,(L-2)!} = \frac{L \bullet (L-1)}{2} \qquad (4)$$

where L is the number of labor in the group. In addition, a possible k-nary cooperation, $k > 2$ within the group can be treated as multiple pairwise ones.

Lemma 1. The *overhead of interpersonal coordination* h in a multi-person project ($L \geq 2$) is proportional to both the number of pairwise relations n and the average interpersonal coordination rate spent in each pair of coordinations r, for example:

$$h = r \bullet n$$
$$= r \bullet \frac{L(L-1)}{2} \qquad (5)$$

where multiple personal relations in the project can be treated as the combinations of multiple pairwise relations.

Lemma 1 indicates that the interpersonal coordination overhead h is a function of r and L (i.e., $h(r, L)$), which represents the efficiency of the transformation between labor and time in a cooperative project. For a given r for a cooperative project, the more the persons involved in a cooperative project, the faster the overhead spent in coordination increasing exponentially. For instance, according to Eq. 5, if there are three persons, i.e. $L = 3$, in a cooperative project where $r = 0.4$, the interpersonal overhead $h = 1.2$; for $L = 10$, $h = 18$; and for $L = 1,000$, $h = 199,800$. Obviously, $h = 0$ if a project is with only one person.

Typical overheads, $h(r, L)$, for interpersonal coordination are provided in Table 2 determined by Eq. 5. With the data derived in Table 2, Lemma 1 can be illustrated by the 13 curves as shown in Fig. 1, where the first curve $h(0.001, L)$ is very close to zero. It is noteworthy that Lemma 1 is a generic model that is valid for the domains $0 < r \leq 100\%$ and $1 \leq L \leq \infty$ for any cooperation project.

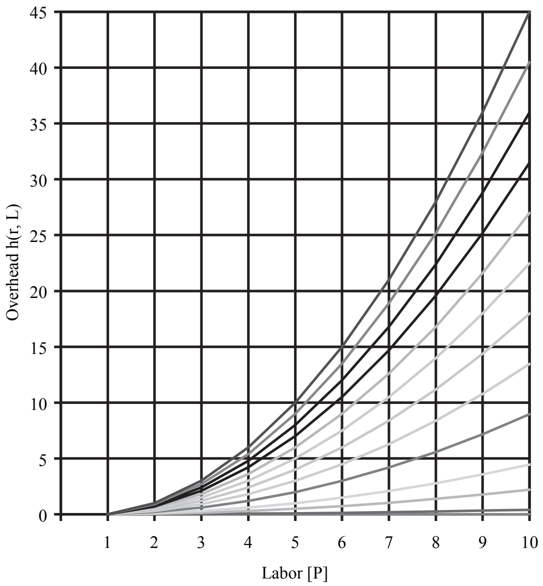
The Natural of Cooperative Work

According to Lemma 1, Eqs. 1 and 5, the inherent nature of cooperative work and the generic form of the actual workload in a cooperative project can be revealed by the following theorem.

Table 2. Overhead of interpersonal coordination h(r, L)

L (P)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
n	0	1	3	6	10	15	21	28	36	45
r	**h(r, L)**									
0.001	0	0.001	0.003	0.006	0.01	0.015	0.021	0.028	0.036	0.045
0.01	0	0.01	0.03	0.06	0.1	0.15	0.21	0.28	0.36	0.45
0.05	0	0.05	0.15	0.3	0.5	0.75	1.05	1.4	1.8	2.25
0.1	0	0.1	0.3	0.6	1	1.5	2.1	2.8	3.6	4.5
0.2	0	0.2	0.6	1.2	2	3	4.2	5.6	7.2	9
0.3	0	0.3	0.9	1.8	3	4.5	6.3	8.4	10.8	13.5
0.4	0	0.4	1.2	2.4	4	6	8.4	11.2	14.4	18
0.5	0	0.5	1.5	3	5	7.5	10.5	14	18	22.5
0.6	0	0.6	1.8	3.6	6	9	12.6	16.8	21.6	27
0.7	0	0.7	2.1	4.2	7	10.5	14.7	19.6	25.2	31.5
0.8	0	0.8	2.4	4.8	8	12	16.8	22.4	28.8	36
0.9	0	0.9	2.7	5.4	9	13.5	18.9	25.2	32.4	40.5
1.0	0	1	3	6	10	15	21	28	36	45

Figure 1. Overhead of interpersonal coordination when r ∈ {0.001 ... 1}



Theorem 1. The 1st law of abstract work organization states that the *actual workload* W of a cooperative project is a function of the average interpersonal coordination rate r and the number of labor L in the project, for example:

$$\begin{aligned} W &= L \bullet T \\ &= L \bullet T_1(1+h) \\ &= W_1(1+h) \\ &= W_1(1 + r \bullet \frac{L(L-1)}{2}) \quad [\text{PM}] \end{aligned} \tag{6}$$

where T_1 is the *indicational duration* needed to complete the work by only one person, and W_1 is the *ideal workload* without the interpersonal overhead h or that of a single person project.

This is the first fundamental finding on laws of rational work organization. Theorem 1 reveals that the ideal workload W_1 defined in Eq. 6 is a special case where the project only involves one person ideally and therefore there is no interpersonal coordination overhead ($h = 0$). Although, some other types of overhead may exist in various projects, $h(r, L)$ is the unique property found only in cooperative projects.

The structures of work organizations and allocations (Wang, 2005a, 2005b) and their effects on the workload model as given in Theorem 1 are analyzed next. Major methods of work organizations are the series and parallel structures, as well as their combinations known as the hybrid structure.

Definition 5. A *serial work organization* is a form of work allocation in which a given workload is decomposed into a series of units/processes and each unit/process is allocated to a person or a subgroup.

Lemma 2. The output work of a serial work organization W_{os} with n units/processes equals to the minimum work done by the least capable unit W_{min}, for example:

$$W_{os} = min(W_i \mid 1 \le i \le n) = W_{\min} \tag{7}$$

Lemma 2 shows that the capacity of a system with serial work organization is determined by the least capable unit W_{min} known as the *bottleneck*.

Definition 6. A parallel work organization is a form of work allocation in which a given work is done repetitively or jointly by multiple persons or subgroups.

Lemma 3. The output work of a parallel work organization W_{op} is equal to the sum of work done by each of the n unit W_i, for example:

$$W_{op} = \sum_{i=1}^{n} W_i \qquad (8)$$

Lemma 3 shows that the capacity of a system with parallel work organization may be dominated by the most capable unit W_{max} known as the *dominator*, whose capacity is greatly larger than those of the remainder.

It is noteworthy that the generic cooperative workload model developed in Theorem 1 is development-cycle/structure independent, because it is only a function of $W(r; L)$. The theory fits all three forms of system organizations in serial, parallel, and hybrid structures at both the unit/process level or the whole project level, because a hybrid structure of work organization can be analyzed segmentally where each segment is a simple serial or parallel structure.

In the context of software engineering, since a software project organized by any process model falls into one of the three basic system structures, the project as a whole or as a set of segmented processes obeys the same rule. Theorem 1 is also applicable to any real-world instance or specific case that uses the waterfall model, incremental model, or process models because various model adoption may only change the instance values of the interpersonal cooperation rate r rather than the law itself. In addition, the following empirical observations and heuristic principles in software and system engineering such as: (a) The Brooks' principle that states "Adding people in a late project make it later (Brooks, 1975)," and (b) The Schonberger's observation on "Why projects are always late (Roberts et al., 1991)," are specific evidences supporting the generic work organization theory.

LAWS OF COOPERATIVE WORK ORGANIZATION

With the modeling and analyses of the cooperative work in the preceding sections, the behaviors and laws of cooperative workload, labor allocation, and the interchangeability of labor and time can be formally analyzed in this section.

The Law of Incompressibility of Workload

Observing Theorem 1, it can be seen that the ideal workload W_1 of a project is the minimum workload in cooperative tasks, and it cannot be reduced no matter how many persons are involved via any kind of labor allocation.

Theorem 2. The second law of abstract work organization, the *law of incompressibility of workload*, states that a given workload W_1 in a cooperative work cannot be compressed by any kind of labor allocation, for example:

$$W \geq W_1 = W_{min} \qquad (9)$$

and in the simplest case when there is only one person involved, the minimum workload $W = W_1 = W_{min}$ may be reached.

Proof: According to Eq. 6, $W = W_1(1+h)$. (a) In a multi-person project, because $h > 0$ by any kind of labor allocation, therefore $W > W_1$. (b) In a single-person project, since $h = 0$, therefore, $W \geq W_1 = W_{min}$.

Theorem 2 indicates that cooperation in a multi-personal project may reduce the duration of the project, but it cannot reduce the total workload because the minimum workload for a given project is reached at $W_{min} = W_1$. In other words, although labor and time may be interchangeable, the minimum workload for a given project is a constant. Therefore, the total workload in any type of cooperative labor allocation will be larger than the minimum.

The Law of Interchangeability Between Labor and Time

On the basis of Theorems 1 and 2, the mathematical model of the interchangeability between labor and time can be formally derived as follows.

Theorem 3. The 3rd law of abstract work organization, the *law of interchangeability of labor and time*, states that, for a given workload W, labor L and duration T are transformable under the following condition:

$$\begin{aligned} T &= \frac{W}{L} \\ &= \frac{W_1}{L}(1 + r \bullet \frac{L(L-1)}{2}) \\ &= \frac{W_1}{L}(\frac{1}{2}rL^2 - \frac{1}{2}rL + 1) \\ &= \frac{1}{2}W_1(rL - r + \frac{2}{L}) \end{aligned} \tag{10}$$

Proof: Solving Eq. 6 for T obtains the previous conclusion.

The third law indicates that the duration of a cooperative project is a function of labor L and the interpersonal cooperation rate r for the given project. In the case where r may be a variable in dynamic project organization, such as in different processes of a software engineering project, each individual process can be treated as a subproject with a constant r, or a mathematical mean of the average r of all processes may be adopted.

The Law of the Shortest Duration of Cooperative Work

Theorem 4. The fourth law of abstract work organization, the *law of* the *shortest duration of cooperative work*, states that there exists the *shortest duration* T_{min} under the *optimum labor allocation* L_0 for a given ideal workload W_1 with a certain interpersonal cooperation rate r, for example:

$$\begin{cases} T_{\min} = \{T \mid L = L_0\} = \frac{1}{2}W_1(rL_0 - r + \frac{2}{L_0}) \quad [M] \\ L_0 = \frac{1.414}{\sqrt{r}},\ r \neq 0 \quad [P] \end{cases}$$

(11)
(12)

Proof: Because $T(r, L)$ as shown in Eq. 10 is a differentiable function on L when r is given for a cooperative project, it reaches the minimum T_{min} when its partial derivative equals to zero, for example:

$$\begin{aligned} \frac{\partial T}{\partial L} &= \frac{\partial}{\partial L}(\frac{1}{2}W_1(rL - r + \frac{2}{L})) \\ &= \frac{1}{2}W_1(r - \frac{2}{L^2}) \\ &= 0 \end{aligned} \tag{13}$$

Eq. 13 yields $r - \frac{2}{L^2} = 0$, for example, the optimal labor allocation is:

$$\begin{aligned} L_0 &= \sqrt{\frac{2}{r}} \\ &= \frac{1.414}{\sqrt{r}}, r \neq 0 \end{aligned}$$

This is the second fundamental finding on rational work organization(Wang, 2007). Theorem 4 reveals, for the first time, that the optimal labor allocation in engineering organization *is not* related to the *size* or the ideal workload of a given project as conventional empirical studies suggested. Surprisingly, it is merely determined by the interpersonal cooperation rate for the project. The fourth law was hardly realized in empirical studies in management science and system engineering (Brooks, 1975; Klir, 1992) because of the vital need for a long chain of insightful reasoning that seamlessly transforms Eq. 1 through Eq. 12.

Although other factors as identified earlier may influence the shortest duration T_{min} in a certain project, Theorem 4 provides insight on cooperative work allocation out of all the trivial factors that have hidden the key truth of work organization for decades since the establishment of management science (Tayler, 1911) and system science (Klir, 1992).

As a result of a complicated long-chain reasoning, Theorems 1 through 4 reveal and prove mathematically that the existence and predictability of the minimum of project duration determined by the best labor allocation under certain group cooperation rate r. Although, there were empirical observations on the minimum, such as Brooks' work (1975), rigorous mathematical explanation has not been created for this important phenomenon in management science (Tayler, 1911), system engineering (Klir, 1992), and operations theories (Schmenner and Swink, 1998).

It is noteworthy that the same cooperation laws and mathematical formulae may be used at subproject or individual process level as well as at the whole project level. In the former case, the labor L does not necessarily need to be treated as a constant in the entire lifecycle of the given project. There are two ways to deal with L's flexibility: (a) Whenever L needs to be different in a certain process of a project, the workload of this process may be recalculated by the same law in the same mathematical form. (b) In the panning phase, L may be deemed as an average of labor allocations in the entire

Table 3. Actual time T(r, L) and actual workload W(r, L) distribution

L (P)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
T(0) (M)	10	5	3.33	2.5	2	1.67	1.43	1.25	1.11	**1**
E(0) (PM)	10	10	10	10	10	10	10	10	10	10
T(0.001)	10	5.01	3.34	2.52	2.02	1.7	1.46	1.29	1.15	**1.05**
E(0.001)	10	10.02	10.03	10.08	10.1	10.2	10.22	10.32	10.35	10.5
T(0.01)	10	5.05	3.4	2.65	2.2	1.92	1.73	1.6	1.51	**1.45**
E(0.01)	10	10.1	10.2	10.6	11	11.52	12.11	12.8	13.59	14.5
T(0.05)	10	5.25	3.8	3.25	3	**2.92**	2.93	3	3.11	3.25
E(0.05)	10	10.5	11.4	13	15	17.52	20.51	24	27.99	32.5
T(0.1)	10	5.5	4.29	**4**	4	4.18	4.43	4.75	5.11	5.5
E(0.1)	10	11	12.87	16	20	25.1	31	38	46	55
T(0.2)	10	6	**5.28**	5.5	6	6.68	7.44	8.25	9.1	10
E(0.2)	10	12	15.84	22	30	40.1	52.1	66	81.9	100
T(0.3)	10	6.5	**6.27**	7	8	9.19	10.44	11.75	13.1	14.5
E(0.3)	10	13	18.81	28	40	55.14	73.08	94	117.9	145
T(0.4)	10	**7**	7.26	8.5	10	11.69	13.44	15.25	17.09	19
E(0.4)	10	14	21.78	34	50	70.14	94.08	122	153.8	190
T(0.5)	10	**7.5**	8.25	10	12	14.2	16.45	18.75	21.1	23.5
E(0.5)	10	15	24.75	40	60	85.2	115.2	150	189.9	235
T(1)	**10**	10	13.2	17.5	22	26.72	31.46	36.25	41.1	46
E(1)	10	20	39.6	70	110	160.3	220.2	290	369.9	460

lifecycle of the project.

A set of typical data between actual duration and actual workload against different labor allocations, subjected to the ideal workload W_1 = 10PM, are shown in Table 3. Any other specific cases can be determined by applying Eqs. 6, 11, and 12.

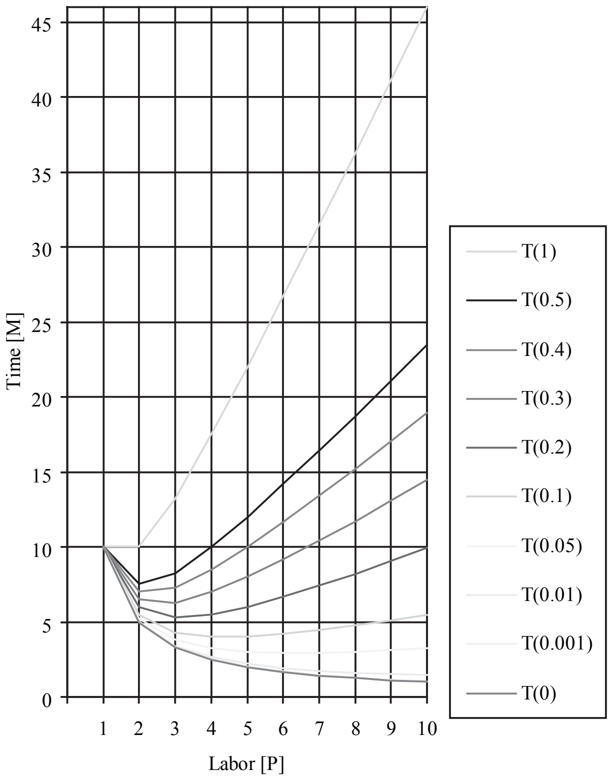
In Table 3, the optimum labor allocation L_0 for each $T(r; L)$ curve is shaded where T reaches its minimum. The curves and trends of actual project durations against different labor allocations are illustrated in Figure 2 known as the *Pigeon Diagram*. The curves indicate that a given optimum labor allocation L_0 for each curve will determine a certain minimum on the curve corresponding to the shortest project duration T_{min}.

Example 1. Assume the ideal workload of a cooperative project is expected to be W_1 = 10PM and the organization has 10 persons available. According to the fourth law, determine the optimum allocation of labor L_0 and the shortest expected duration T_{min} for this project when the average interpersonal coordination rate r = 10%.

Applying Theorem 4, we obtain:

$$\begin{aligned} L_0(r) &= \frac{1.414}{\sqrt{r}} \\ &= \frac{1.414}{\sqrt{0.1}} \\ &= 4.47 \\ &\approx 5.0 \text{ [P]} \end{aligned}$$

Figure 2. The Pigeon Diagram: Actual time against number of labors (W_1 = 10PM)



Replacing L_0 in Eq. 11 with the instantiation value $L_0(r)$, the shortest duration of the project can be determined as follows:

$$\begin{aligned} T_{\min} &= \frac{1}{2}W_1(rL_0 - r + \frac{2}{L_0}) \\ &= 0.5 \bullet 10 \bullet (0.1 \bullet 5 - 0.1 + 2/5) \\ &= 5.0 \bullet 0.8 \\ &= 4.0 \text{ [M]} \end{aligned}$$

The previous solutions indicate that, for a project with an expected 10PM workload by one person, the optimum labor allocation and shortest possible duration implemented by a cooperative project are 5.0 persons for 4.0 months, respectively, under r = 10%. This results in a real workload of W = 5.0P • 4.0M = 20.0PM in cooperation.

Example 2. Comparatively reanalyze Example 1 for a given average interpersonal coordination rate r = 50%.

The optimum solution yielded for the same project with expected 10PM workload under r = 50% is to spend 22.5 months by allocating 2.0 persons, which results in a real workload of W = 2.0P • 22.5M = 45.0PM.

On the basis of Theorem 4, an important corollary may be derived below, which clarifies the myth that whether labor L or duration T in a cooperative project is arbitrarily determinable as mentioned in the beginning of this article.

Corollary 1. An optimal work organization must be carried out in the following order for a given cooperative project:

a. To determine the optimum labor allocation L_0 (Eq. 12).
b. To obtain the shortest duration of the cooperative work T_{min} under L_0 (Eq. 11).

This is the third fundamental finding of this work on laws of rational work organization. Corollary 1 indicates that the conventional common sense which believed that labor L or duration T in a cooperative project is arbitrarily determinable (Brooks, 1975; Gantt, 1919; Tayler, 1911) was a risky organizational practice that could easily result in a lot of waste of both resources and time without awareness in large cooperative projects. Further analysis will be shown in Example 3.

Optimization of Project Organization for the Lowest Workload/Cost

On the basis of the cooperation theory and laws presented in preceding sections, a number of decision optimization strategies may be derived toward the objectives of: (a) The best labor allocations/the shortest duration of project; (b) The lowest workload/costs; and (c) The lowest overhead of interpersonal coordination.

Using the data provided in Table 3, the curves of actual workloads with varying coordination rates against different number of labors are illustrated in Figure 3. Observing Figure 3, it can be seen that all curves obey Theorem 1 that states $W_{min} = W_1$. Further, the following corollary can be derived.

Corollary 2. The strategy for optimization of a cooperative project for the *lowest cost* is to set the project at $W(T_{min}, L_0)$. Otherwise, the waste of effort ΔW can be determined as:

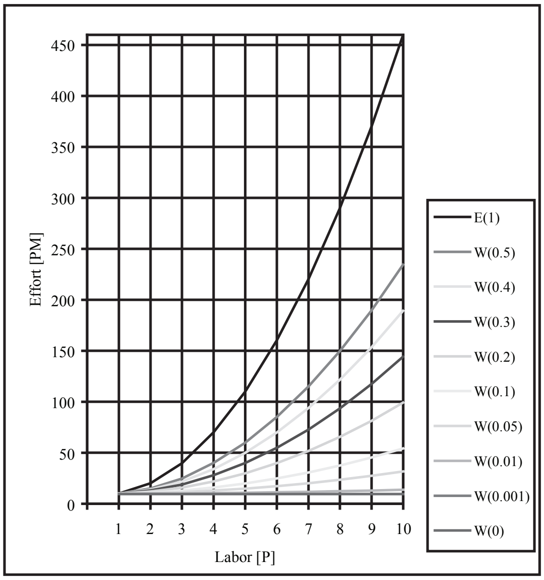
$$\Delta W = W - W_{\min} \text{ [PM]} \qquad (14)$$

where W is the realized workload due to a non-optimal work allocation.

Example 3. In Example 1, the optimal work allocation has been determined as $W_{\min} = L_0 \bullet T_{\min} = 5.0 \bullet 4.0 = 20.0$ [PM] . When the number of persons for this project is *subjectively* allocated by L = *9.0*P, what will be the amount of the real workload W resulted? How much effort would be wasted due to this *non-optimal* labor and time allocation?

According to Eqs. 10 and 6, the duration T and the real workload W of this project for given L = *9.0*P and r = 0.1 can be determined, respectively, as follows:

Figure 3. Actual effort against number of labors when W_1 = 10PM



$$\begin{aligned} T &= \frac{1}{2}W_1(rL - r + \frac{2}{L}) \\ &= 0.5 \bullet 10 \bullet (0.1 \bullet 9.0 - 0.1 + 2/9.0) \\ &= 5.0 \bullet 1.02 \\ &= 5.1 \text{ [M]} \end{aligned}$$

$$\begin{aligned} W &= L \bullet T \\ &= 9.0 \bullet 5.1 \\ &= 45.9 \text{ [PM]} \end{aligned}$$

Due to the *non-optimal* labor and time allocation in this cooperative project, the effort wasted, ΔW, can be expected as follows:

$$\begin{aligned} \Delta W &= W - W_{\min} \\ &= 45.9 - 20.0 \\ &= 25.9 \text{ [PM]} \end{aligned}$$

This case study indicates that a complicated engineering project, particularly in software engineering, may be easily turned to a failure due to bad organizational decisions with a non-optimal labor allocation as stated next.

Theorem 5. The risks due to irrational decisions of work organization are proportional to the coordination rate *r* in a coordinative project. That is, the higher the *r*, the higher the risk under non-optimal labor allocation.

THE EXCHANGEABILITY AND CONSTRAINTS BETWEEN LABOR AND TIME IN WORK ORGANIZATION

According to Theorem 1, the trade-off between labor and time is possible under certain conditions. This section analyzes the equivalence and the exchange rate between labor and time in cooperative work organization, as well as their constraints and conditions.

Definition 7. The *maximum gain of time* ΔT of a multi-person project is the difference between the time needed when only one person is allocated for the project and the shortest time T_{min} when labor is optimally allocated at L_0, for example:

$$\Delta T = T_1 - T_{min} \tag{15}$$

Definition 8. The *maximum increment of labor* ΔL of a multi-person project is the difference between the optimum allocated number of persons L_0 and the smallest group where L_1 =1, for example:

$$\Delta L = L_0 - L_1, \quad L_1 \equiv 1 \tag{16}$$

Theorem 6. The *exchange rate from labor to time* $\gamma_{L\sim T}$ in a cooperative work organization is determined by the ratio between the increment of time ΔT and the increment of labor ΔL, for example:

$$\begin{aligned}\gamma_{L\sim T} &= \frac{\Delta T}{\Delta L} \\ &= \frac{T_1 - T_{min}}{L_0 - L_1} \quad [\text{M/P}]\end{aligned} \tag{17}$$

The physical meaning of Eq. 17 is how many expected months may be gained or shortened in the schedule of a cooperative project by adding per labor into the project. It also explains how many months may be delayed if a person is withdrawn from the project.

Example 4. The exchange rate from labor to time $\gamma_{L\sim T}$ as given in Example 1 can be determined as follows:

$$\begin{aligned}\gamma_{L\sim T} &= \frac{T_1 - T_{min}}{L_0 - L_1} \\ &= \frac{10-4}{5-1} \\ &= 1.5 \quad [\text{M/P}]\end{aligned}$$

The result shows that around the optimum labor allocation point, the increment of persons is most effective to progress a project. For this given example, the increment of each person can reduce the project duration for 1.5 months.

Theorem 7. The *exchange rate from time to labor* $\gamma_{T\sim L}$ in a cooperative work organization is determined by the ratio between the increment of labor ΔL and the increment of time ΔT, for example:

$$\begin{aligned}\gamma_{L\sim T} &= \frac{\Delta L}{\Delta T} \\ &= \frac{L_0 - L_1}{T_1 - T_{\min}} \quad [\text{P/M}]\end{aligned} \tag{18}$$

The physical meaning of Eq. 18 is how many persons' work is equivalent to a monthly increase/decrease in the project duration.

Example 5. The exchange rate from time to labor $\gamma_{T\sim L}$ as given in Example 1 can be determined as follows:

$$\begin{aligned}\gamma_{L\sim T} &= \frac{L_0 - L_1}{T_1 - T_{min}} \\ &= \frac{5-1}{10-4} \\ &= 0.67 \quad [\text{P/M}]\end{aligned}$$

The result shows that, in the most effective case, an action to add an extra month in the schedule is equivalent to the reducing of 0.67 person in the whole project lifecycle.

Comparing Eqs. 17 and 18, it can be observed that the two exchange rates are reciprocal. This leads to the following corollary.

Corollary 3. Labor and time are bidirectionally interchangeable or transformable in cooperative work organization under the constraints of Theorems 1 through 6, for example:

$$\gamma_{L\sim T} = \gamma{-1}_{L\sim T} \tag{19}$$

Theorems 1 through 7, Laws 1 through 4, and related corollaries have answered the fundamental questions in cooperative work organization raised in the beginning of this article by rigorous reasoning and inferences. As a result, the insightful nature and inherent mechanisms of the problems in cooperation work organization are systematically revealed and explained.

CONCLUSION

This article has systematically studied the nature of cooperative work and their mathematical models. The mathematical models created in this article have formed a foundation for analyzing the mechanisms and behaviors of human cooperative work and their organization. The article has revealed the transformability between labor and time in cooperative work and the exchange rates between them in engineering organization. A set of work organization laws have been derived that enables quantitative and rigorous predication of the optimal labor allocation, the shortest project duration, and the minimum effort in cooperative project organization.

The article has addressed an age-long problem on cooperative project and group organization and optimization across many disciplines such as management science, operations theories, system science, software engineering, economics, and sociology. Conventional work has been focused on empirical studies of project planning and scheduling, and the inherent nature of the problem was hidden by too many trivial factors. This is the first time that it has been revealed that the interpersonal cooperation rate in group is the black hole that has resulted in the failures of so many large-scale projects due to the exponential growing of unexpected actual workload under non-optimal labor and work allocation. Based on the Wang's laws of rational work organization theories, a wide range of applications in optimal engineering organization have been studied, such as the decision optimization strategies in engineering cooperation, and the determination of the best labor allocation, the shortest duration, and the lowest effort (cost) in project organization.

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