

On Cognitive Foundations of Creativity and the Cognitive Process of Creation

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ABSTRACT

Creativity is a gifted ability of human beings in thinking, inference, problem solving, and product development. A creation is a new and unusual relation between two or more objects that generates a novel and meaningful concept, solution, method, explanation, or product. This article formally investigates into the cognitive process of creation and creativity as one of the most fantastic life functions. The cognitive foundations of creativity are explored in order to explain the space of creativity, the approaches to creativity, the relationship between creation and problem solving, and the common attributes of inventors. A set of mathematical models of creation and creativity is established on the basis of the tree structures and properties of human knowledge known as concept trees. The measurement of creativity is quantitatively analyzed, followed by the formal elaboration of the cognitive process of creation as a part of the Layered Reference Model of the Brain (LRMB).

Keywords: Cognitive Informatics, Brain, LRMB, Cognitive Model, Mathematical Model, Cognitive Processes, RTPA, Creation, Creativity, Computational Intelligence, Denotational Mathematics, Abstract Intelligence, Cognitive Computing, AI

INTRODUCTION

Creativity is a gifted ability of human beings in thinking, inference, problem solving, and product development (Beveridge, 1975; Csikszentmihalyi, 1996; Holland, 1986; Matlin, 1998; Smith, 1995; Sternberg & Lubart, 1995; Wang et al., 2006; Wilson & Keil, 1999). Human creativity may be classified into three categories known as the abstract, concrete, and art creativities. A scientific (abstract) creation is usually characterized by a free and unlimited creative environment where the goals and paths for such a creation is totally free and unlimited; while an engineering (concrete) creation is characterized by a limited creative environment where a creative problem solving is constructed by a certain set of goals, paths, and available conditions. The third form of creation is the art (empirical) creation that generates a novel artifact that attracts human sensorial attention and perceptual satisfactory.

DOI: 10.4018/jcini.2009062301

Creativity has been perceived diversely and controversially in psychology, intelligence science, and cognitive science (Csikszentmihalyi, 1996; Guilford, 1967; Leahey, 1997; Mednich & Mednich, 1967; Matlin, 1998; Sternberg & Lubart, 1995; Wallas, 1926; Wang et al., 2009a, 2009b). Creativity may be treated as a form of art that generates unexpected results by unexpected paths and means. It may also be modeled as a scientific phenomenon that generates unexpected results by purposeful pursuits. In 1998, Matlin perceived that creativity is a special case of problem solving (Matlin, 1998). From this perspective, he defined creativity as a process to find a solution that is both novel and useful. However, problem solving often deals with issues for a certain goal with unknown paths. Therefore, creation is much more divergent that deals with issues of both unknown goals and unknown paths for a problem under study.

The nature of creations is a new and unusual relation between two or more objects that generates a novel and meaningful concept, solution, method, explanation, or product. This article investigates into the cognitive process of creation and creativity as a higher-layer life function. Cognitive foundations of creativity are explored on such as the space of creativity, the approaches to creativity, the relationships of creation and problem solving, and the attributes of creative researchers. A set of mathematical models of creation and creativity is developed by studying the tree structures and properties of human knowledge known as concept trees. On the basis of the concept tree, the measurement of creativity is quantitatively analyzed. The cognitive process of creation is rigorously elaborated with Real-Time Process Algebra (RTPA) (Wang, 2002a, 2007a, 2008a, 2008e), which provides a formal explanation of human creativity.

COGNITIVE FOUNDATIONS OF CREATIVITY²

Human creativity as a gifted ability is an intelligent driving force that brings something into existence.

Definition 1. *Creativity is the intellectual ability to make creations, inventions, and discoveries that brings novel relations and entities or unexpected solutions into existence.*

Definition 2. *A creation is a higher cognitive process of the brain at the higher cognitive layer that discovers a new relation between objects, attributes, concepts, phenomena, and events, which is original, proven true, and useful.*

Wallas identified five stages in a creative process (Wallas, 1926) as follows: (1) *preparation*, (2) *incubation*, (3) *insight*, (4) *evaluation*, and (5) *elaboration*. Csikszentmihalyi pointed out that creativity can best be understood as a confluence of three factors: a *domain* that consists of a set of rules and practices; an *individual* who makes a novel variation in the contents of the domain; and a *field* that consists of experts who act as gatekeepers to the domain, and decide which novel variation is worth adding to it (Csikszentmihalyi, 1996).

Various creativities and creation processes may be identified such as free/constrained creativity, analytic/synthetic creativity, inference-based creativity, problem-solving-based creativity, and scientific/ technological/art creativity. The entire set of creativities can be classified into three categories according to their creation spaces, approaches, and problem domains as summarized in Table 1.

Table 1. Taxonomy of creativity and creation

No.	Category	Type of creation	Description	Reference
1	Creation space	Free	A creation process with an unlimited creation space S_c , which is determined by unconstrained sets of alternatives N_a , paths N_p , and goals N_g .	Def. 4
2		Constrained	A creation process with a limited creation space S'_c where one or more conditions such as the goals N'_g , paths N'_p , or alternatives N'_a , are limited.	Def. 5
3	Approach	Analytic	A top-down creation process that discovers a novel solution to a given problem by deducing it to the subproblem level where new or existing solutions may be found.	Def. 7
4		Synthetic	A bottom-up creation process that discovers a novel solution to a given problem by inducting it to a superproblem where new or existing solutions may be found.	Def. 8
5		Inference-based	An abstract creativity based on the deductive, inductive, abductive, and analogy inference methodologies.	Def. 9
6		Problem-solving-based	A novel solution for a given problem by creative goals and/or creative paths.	Def. 15
7	Domain	Scientific (abstract)	A free and unlimited creative environment where the goals and paths for such a creation is totally free and unlimited.	Section 1
8		Technological (concrete)	A limited creative environment where a creative problem solving is constructed by a certain set of goals, paths, and available conditions.	Section 1
9		Art (empirical)	A free and unlimited creative environment where a novel artifact is generated that attracts human sensorial attention and perceptual satisfactory	Section 1

Definition 3. A creation space Θ is a Cartesian product of a nonempty set of baseline alternatives A , a nonempty set of paths P , and a nonempty set of goals G , i.e.:

$$\Theta \triangleq A \times P \times G \quad (1)$$

where \times represents a Cartesian product.

The Space of Creativity

On the basis of the creation space, the nature of free and constrained creativities can be explained.

Definition 4. A free creativity is a creation process with an unlimited creation space S_c , $S_c \subseteq \Theta$, which is determined by unconstrained sets of alternatives N_a , paths N_p , and goals N_g , i.e.:

$$\begin{aligned}
 S_c &\triangleq N_a \bullet N_p \bullet N_g \\
 &= \# A \bullet \# P \bullet \# G
 \end{aligned}
 \tag{2}$$

where # is the cardinal calculus that counts the number of elements in a given set.

Eq. 2 indicates that the creative space of a free creation may very easily turn to be infinitive, because N_a , N_p , and N_g can be extremely large. Therefore, the cost or difficulty of creation is often extremely high. That is, only mechanical and exhaustive search is insufficient in most cases for potential creations and discoveries, if it is not directed by heuristic and intelligent vision. In other words, creations and discoveries are usually achieved only by chance of purposeful endeavors of prepared minds, where an appreciation of highly unexpected result is always prepared. This is also inline with the empirical finding of Pasteur as he stated that “Creation always favorites prepared minds (Beveridge, 1975).”

Definition 5. A constrained creativity is a creation process with a limited creation space S'_c , $S'_c \subseteq S_c \subseteq \Theta$, where one or more conditions such as the goals N'_g , paths N'_p , or alternatives N'_a are limited, i.e.:

$$\begin{aligned}
 S'_c &\triangleq N'_a \bullet N'_p \bullet N'_g \\
 &= \# A' \bullet \# P' \bullet \# G', \quad A' \subset A \wedge P' \subset P \wedge G' \subset G
 \end{aligned}
 \tag{3}$$

Usually, a scientific and art creation is characterized as a free creation process, while an engineering creation is featured as a constrained creation process.

Approaches to Creativity

A variety of typical and sometimes controversial approaches to creation have been identified in literature, such as divergent production (Guiford, 1967), remote association test (Mednich & Mednich, 1967), analysis/synthesis (Wang et al., 2006), and inferences (Wang, 2007c).

Wallas (1926), Beverage (1957), and Smith (1995) pointed out an important phenomenon in human creativity known as *incubation*.

Definition 6. Incubation is a mental phenomenon that a breakthrough in problem solving may not be achieved in a continuous intensive thinking and inference until an interrupt or interleave action is conducting, usually in a relax environment and atmosphere.

The cognitive mechanism of incubation can be explained by the subconscious processes of the brain related to thinking and inference, such as perception, imagination, and unintentional search, which are involved in complex thinking and long chains of inferences. Whenever there is an impasse, incubation may often lead to a creation under the effect of active subconscious processes. Incubation has been observed to play an active role in the creation process.

The approaches to creativity can be categorized into the analytic, synthetic, and inference approaches as described below.

Definition 7. *An analytic creativity is a top-down creation process that discovers a novel solution to a given problem by deducing it to the subproblem level where new or existing solutions may be found.*

Definition 8. *A synthetic creativity is a bottom-up creation process that discovers a novel solution to a given problem by inducing it to a superproblem where new or existing solutions may be found.*

Definition 9. *An inference creativity is an abstract creativity based on the deductive, inductive, abductive, and analogy inference methodologies.*

The inference methodologies as a fundamental approach to creativity have been formally studied in (Wang, 2007c).

Creation vs. Problem Solving

As creativity is a novel or unexpected solution to a given problem, a creation may be perceived as a special novel solution where the problem, goal, or path is usually unknown. Therefore, the study of the generic theory of creativity can be reduced to the theory of problem solving (Wang & Chiew, 2009). The theoretical framework of problem solving can be modeled as follows.

Definition 10. *A problem solving is a cognitive process of the brain that searches or infers a solution for a given problem in the form of a set of paths to reach a set of given goals.*

Definition 11. *Assuming the layout of a problem solving is a function $f : X \rightarrow \dots \rightarrow Y$, the problem ρ is the domain of f , X , in general, and a specific instance x , $x \in X$, in particular, i.e.:*

$$\rho \triangleq (X \mid f : X \rightarrow \dots \rightarrow Y) \quad (4)$$

Eq. 4 denotes that, in problem solving, a problem ρ is the fix point of a function in general, and the input of the function in particular. The former is the broad sense of a problem, and the latter is the narrow sense of a problem.

Problem solving is a process that seeks the generic function for a layout of problem, and determines its domain and codomain. Then, a solution in problem solving can be perceived as a concrete instance of a given function for the layout of the problem.

According to Definition 11, there are two categories of problems in problem solving: (a) The *convergent* problem where the goal of problem solving is given but the path of problem solving is unknown; and (b) The *divergent* problem where the goal of problem solving is unknown and the path of problem solving are either known or unknown.

Definition 12. *A goal G in problem solving is a terminal result Y of satisfactory in the creation space S_c of the problem ρ , which deduce X to Y by a sequence of inference in finite steps, i.e.:*

$$G \triangleq (Y \mid X \rightarrow \dots \rightarrow Y), \quad G \in \Theta \quad (5)$$

Definition 13. A path P in problem solving is a 3-tuple with a nonempty finite set of problem inputs X , a nonempty finite set of traces T , and a nonempty finite set of goals G , i.e.:

$$\begin{aligned} P &\triangleq (X, T, G) \\ &= X \times T \times G \end{aligned} \quad (6)$$

where the traces T is a set of internal nodes or possible subpaths that leads to the solution.

Definition 14. A solution to a given problem ρ is a selected relation or function, S , which is an instance of the solution paths in P , i.e.:

$$S \triangleq (X, T, G) \subseteq P, \quad X, T, G \neq \emptyset \quad (7)$$

The solutions S and paths P in problem solving as modeled in Definition 14 can be illustrated in Figure 1.

Theorem 1. The polymorphic solutions state that the solution space SS , $SS \subseteq \Theta$, of a given problem ρ is a product of the numbers of problem inputs N_x , traces N_t , and goals N_g , i.e.:

$$\begin{aligned} SS &\triangleq N_x \bullet N_t \bullet N_g \\ &= \#X \bullet \#T \bullet \#G \end{aligned} \quad (8)$$

The polymorphic characteristic of the solution space contributes greatly to the complexity of problem solving and creations. It is noteworthy that the path $p(x, t, g) \in P$ in Definition 13 can be a simple or a complex function. A complex function that mapping a given problem into a solution goal may be very complicated depending on the nature of the problem.

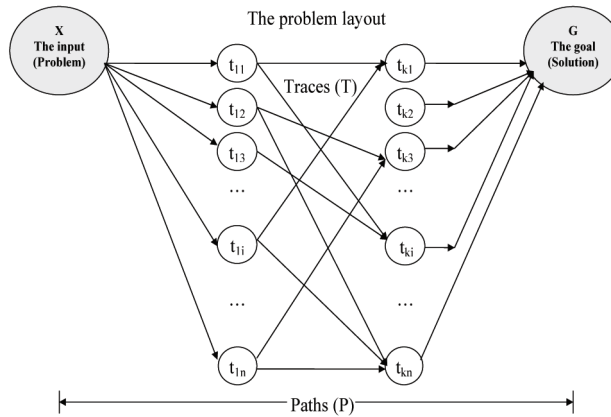
According to Definition 13, in case $\#X = 0$, $\#G = 0$, or $\#T = 0$, there is no solution for the given problem. For a convergent problem, i.e. $\#G = 1$, the number of possible solutions $SS = \#X \bullet \#T$.

Definition 15. A creation C is a novel and unexpected solution S_ρ , which is a subset of the entire set of SS that meets the criteria of novelty, originality, and utility, or the originality of the creation is true, i.e. $O = 1$, i.e.:

$$\begin{aligned} C &\triangleq (S_0 \mid S_0 \subseteq P \subseteq SS \wedge O = 1) \\ &= X_0 \times T_0 \times G_0, \quad X_0 \subseteq X \wedge T_0 \subseteq T \wedge G_0 \subseteq G \wedge O = 1 \end{aligned} \quad (9)$$

It is noteworthy that, although a creation C is a subset of the entire solutions S for a given problem, it is always the unknown and novel subset, which extends the entire solution set.

According to Definition 15, a creation is a search for the unknown goals, unknown paths, or both under a given problem or a set of coherent problems. Therefore, creations can be classified into the categories of goal-driven, method-driven, and problem-driven. Among them, the

Figure 1. A solution in creation trace T and problem solving

problem-driven creation is a fully open process because both goals and paths are unknown for the given problem.

Attributes of Inventors

A number of typical attributes sharing by inventors have been studied. In his book on *The Art of Scientific Investigation*, Beveridge (1957), a professor at Cambridge University, thought that the research scientists are fortunate in that in their work they can find something to give meaning and satisfaction to life. Beveridge identified a set of attributes required for researchers and inventors, such as enterprise, curiosity, initiative, readiness to overcome difficulties, perseverance, a spirit of adventure, a dissatisfaction with well-known territory and prevailing ideas, and an eagerness to try his own judgment, intelligence, imagination, internal drive, willingness to work hard, perseverance and tenacity of purpose (Beveridge, 1957). In the inventive theory of creation in psychology, Sternberg and Lubart's (1995) elicited the following set of attributes of inventors in psychology, such as intelligence, knowledge, motivation, appreciation, thinking style, and personality. Contrasting the two sets of attributes identifies above, it is interesting to note that the former would have understood scientific creation and invention deeper and with much insight than that of the psychological observations.

Beveridge believed that an insatiable curiosity and love of science are the two most essential attributes of scientists. He pointed out that a good maxim for researchers is look out for the unexpected. He described that creators are those whose imagination are fired by the prospect of finding out something never before found by man, and only for those will succeed who have a genuine interest and enthusiasm for discovery (Beveridge, 1957). Another crucial attribute is perseverance or persistence as Pasteur wrote: "Let me tell you the secret that has led me to my goal. My only strength lies in my tenacity (Dubos, 1950)." Pasteur has also revealed that "In the field of observation, chance favors only the prepared mind."

It is noteworthy that the above investigations into research and researchers have overlooked a more significant attribute for creativity and discovery ability, i.e., *mathematical skills* or the *abstract inference capability*, because mathematics plays the ultimate role of meta-methodology in science and engineering creativities. Actually, mathematical skills and abstraction capabil-

ity are the most important foundation for scientific creation and invention, which enables a scientist to inductively generalize a hypothesis into the maximum scope, usually the infinitive or the universal domain based on limited sample empirical studies and/or mathematical/logical inferences. It is noteworthy that mathematics is the generic foundation of all science and engineering disciplines, as well as all scientific methodologies. To a certain extent, the maturity of a discipline is characterized by the maturity of its mathematical means (Bender, 2000; Zadeh, 1965, 1973; Wang, 2007a, 2008a, 2008b). One of the major purposes of cognitive informatics is to develop and introduce suitable mathematical means into the enquiry of natural intelligence, computational intelligence, cognitive science, and software science. The studies on denotational mathematics (Wang, 2008a, 2008b), such as system algebra (Wang, 2008d), concept algebra (Wang, 2008c), RTPA (Wang, 2002b, 2007a, 2008a, 2008e), and Visual Semantic Algebra (VSA) (Wang, 2009b) are fundamental endeavors towards the formalization of the entities that are conventionally hard-to-be-formalized.

According to cognitive informatics (Wang, 2002a, 2003, 2007b, 2009a; Wang & Wang, 2006; Wang et al., 2006, 2009a, 2009b), significant cognitive attributes related to creativity are those of knowledge organizational efficiency, searching efficiency, abstract ability, appreciation of new relations, curiosity, induction, and categorization, because those identified in the list are fundamental cognitive mechanisms and processes of the brain at the layers of meta-cognition and meta-inference according to the Layered Reference Model of the Brain (LRMB) (Wang et al., 2006), which are frequently used in supporting higher layer cognitive processes.

MATHEMATICAL MODELS OF CREATION AND CREATIVITY

On the basis of the discussions on the cognitive foundations of creativity, a more rigorous treatment of it can be developed in this section on the mathematical models of creation and creativity. The tree structure of human knowledge in term of concept trees and their properties are introduced. Then, a measurement model of creativity is quantitatively established.

The Tree Structure of Human Knowledge

It has been empirically observed that the tree-like architecture is a universal hierarchical prototype of systems across disciplines of not only science and engineering, but also sociology and living systems. The underlying reasons that force systems to take hierarchical tree structures are: a) The complexity of an unstructured system can easily grow out of control; b) The efficiency of an unstructured system can be very low; and c) The gain of system by coordination may diminish when the overhead for doing so is too high in unstructured systems.

An ideal structural form for modeling the knowledge system and creative space of humans is known as the complete tree (Wang, 2007a).

Definition 16. *A complete n -nary tree $T_c(n, N)$ is a normalized tree with N nodes in which each node of T_c can have at most n children, each level k of T_c from top-down can have at most n^k nodes, and all levels have allocated the maximum number of possible nodes, except only those on the rightmost subtrees and leaves.*

It is noteworthy in Definition 16, a tree said to be *complete* means that all levels of the tree have been allocated the maximum number of possible nodes, except those at the leave level and the rightmost subtree. The advantage of complete trees is that the configuration of any complete

n -nary tree $T_c(n, N)$ is uniquely determined by only two attributes: the unified *fan-out* n and the number of leave nodes N at the bottom level. For instance, the growth of a system from complete tree $T_{c1}(n_1, N_1) = T_{c1}(2, 3)$ to $T_{c2}(n_2, N_2) = T_{c1}(2, 7)$ is illustrated in Figure 2.

Theorem 2. *The generic topology of normalized systems states that systems tend to be normalized into a hierarchical structure in the form of a complete n -nary tree.*

Systems are forced to be with tree-like structures in order to maintain equilibrium, evolvability, and optimal predictability. The advantages of tree structures of systems can be formally described in the following corollary.

Corollary 1. *Advantages of the normalized tree architecture of systems are as follows:*

- a. *Equilibrium: Looking down from any node at a level of the system tree, except at the leave level, the structural property of fan-out or the number of coordinated components are the same and evenly distributed.*
- b. *Evolvability: A normalized system does not change the existing structure for future growth needs.*
- c. *Optimal predictability: There is an optimal approach to create a unique system structure $T_c(n, N)$ determined by the attributes of the unified fan-out n and the number of leave nodes N at the bottom level.*

Properties of the Concept Tree of Knowledge Space

Based on the model of the complete tree, the topology of the knowledge space for creation can be denoted as a concept tree with each node of the n -nary complete tree as a concept.

Definition 17. *A concept tree, $CT(n, N)$, is an n -nary complete tree in which all leave nodes N represent a meta-concept, and all remainder nodes beyond the leave level represent superconcepts.*

For instance, a ternary CT, $CT(n, N) = CT(3, 24)$, is shown in Figure 3. Since the CT is a complete tree, when the leaves (components) do not reach the maximum possible numbers, the right most leaves and subtrees of the CT will be left open.

A set of useful topological properties of CT is identified as summarized in the following corollary (Wang, 2007a).

Corollary 2. *An n -nary concept tree $CT(n, N)$ with the total number of leaves nodes N possesses the following properties:*

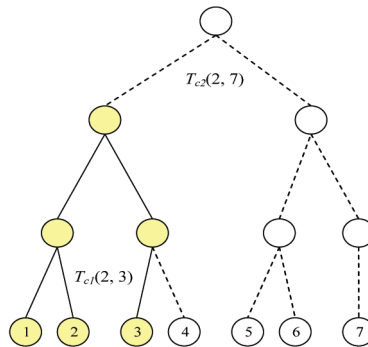
- a. *The maximum number of fan-out of any node \bar{n}_{fo} :*

$$\bar{n}_{fo} = n \quad (10)$$

- b. *The maximum number of nodes at a given level k , n_k :*

$$n_k = n^k \quad (11)$$

Figure 2. Growth of complete binary trees



c. The depth of the CT, d :

$$d = \left\lceil \frac{\log N}{\log n} \right\rceil \tag{12}$$

d. The maximum number of nodes in the CT, N_{CT} :

$$N_{CT} = \sum_{k=0}^d n^k \tag{13}$$

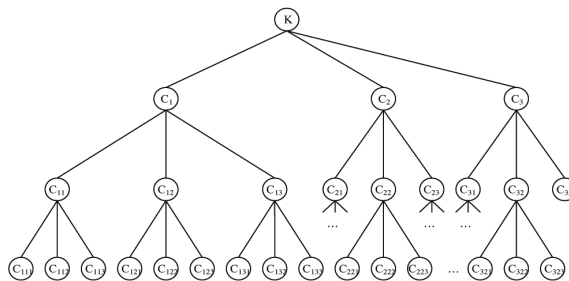
e. The maximum number of meta-concepts (on all leaves) in the CT, N_{max} :

$$N_{max} = n^d \tag{14}$$

f. The maximum number of subtrees (nodes except all leaves) in the CT, N_m :

$$N_m = N_{CT} - N_{max} - 1 = \sum_{k=1}^{d-1} n^k \tag{15}$$

Figure 3. A ternary concept tree $CT(3, 24)$



CT can be used to model and analyze the knowledge space of creativity. It also shows that a well organized knowledge tree in the brain is helpful for creation, because it can greatly reduce the cost for search.

Measurement of Creativity

On the basis of *CT*, a creation is modeled by the relational distances between two or more concepts in the concept tree.

Definition 18. *The relational distance of a creation, δ , is a sum of the distances δ_1 and δ_2 of a pair of concepts or objects c_1 and c_2 to their most closed parent node c_p in a given concept tree *CT*, i.e.:*

$$\begin{aligned}\delta(c_1, c_2) &\triangleq \delta_1 + \delta_2 \\ &= |c_1 \leftrightarrow c_p| + |c_2 \leftrightarrow c_p|\end{aligned}\quad (16)$$

where $\delta_i = |c_i \leftrightarrow c_p|$ denotes the distance between a concept i and its most closed parent concept p shared with another given concept.

According to Definition 18, the minimum creation distance $\delta_{\min}(c_1, c_2) = 2$ when any pair of concepts at the same level of the *CT* under the same parent node.

Definition 18 can be extended to a more general case where multiple concepts are involved in a given *CT* as follows.

Definition 19. *The general relational distance of a creation, δ , is a sum of n , $n > 1$, subdistances δ_i , $1 \leq i \leq n$, between all individual concepts c_i and the most closed parent node c_p in the given knowledge space modeled by a *CT*, i.e.:*

$$\begin{aligned}\delta &\triangleq \sum_{i=1}^n \delta_i \\ &= \sum_{i=1}^n |c_i \leftrightarrow c_p|\end{aligned}\quad (17)$$

Example 1. *Given a knowledge space modeled by a *CT* as shown in Figure 3, any potential pairwise or multiple creation distances can be determined according to Definition 19 as follows:*

$$\begin{aligned}\delta(c_{111}, c_{113}) &= |c_{111} \leftrightarrow c_{11}| + |c_{113} \leftrightarrow c_{11}| = 1 + 1 = 2 \\ \delta(c_{121}, c_{323}) &= 3 + 3 = 6 \\ \delta(c_{111}, c_{113}, c_{121}, c_{323}) &= 3 + (3 - 2) + (3 - 1) + 3 = 9\end{aligned}\quad (18)$$

It is noteworthy that the creativity of a creation is proportional not only to its relational distance, but also to its originality and usefulness.

Definition 20. Assume $O = \{0, 1\}$ is a Boolean evaluation for the false or true originality of a creation, M the total number of nodes at level k in the d level creation space for a given concept tree CT . Then, the extent of creativity C is a product of the creation distance δ , the size of the creation space M , and its originality O , i.e.:

$$\begin{aligned} C &\triangleq (\delta \bullet M) \bullet O \\ &= \delta O \bullet \sum_{i=0}^{d-k} n^i \end{aligned} \quad (19)$$

where n is the fan-out of the given CT .

Example 2. Based on the three solutions as given in Example 1, assume their originalities $O_1 = O_2 = O_3 = 1$, then the creativities of the three solutions can be quantitatively evaluated as follows:

$$\begin{aligned} C_1 &= \delta_1 O_1 \bullet \sum_{i=0}^{d-k_1} n^i = 2 \bullet 1 \bullet \sum_{i=0}^{3-2} n^i = 2 \bullet (1 + 3) = 8 \\ C_2 &= \delta_2 O_2 \bullet \sum_{i=0}^{3-1} n^i = 6 \bullet (1 + 3 + 9) = 78 \\ C_3 &= \delta_3 O_3 \bullet \sum_{i=0}^{3-0} n^i = 9 \bullet (1 + 3 + 9 + 27) = 360 \end{aligned} \quad (20)$$

Corollary 3. The creativity of a creation is proportional to the product of the creative distance and the size of the creation space, subject to a satisfactory originality.

THE COGNITIVE PROCESS OF CREATION

With the cognitive and mathematical models of creation and creativity developed in previous sections, the process model of creation can be formally described in this section.

The Conceptual Model of the Creation Process

On the basis of Definitions 13, 14, and 15, a search-based creation process is modeled as shown in Figure 4, where an informal process of creation is divided into the following six steps: i) To define the problem; ii) To search the solution goals and paths; iii) To generate candidate solutions; iv) To identify and evaluate novel solutions; v) To represent creative solutions; and vi) To memorize creative relations.

It is noteworthy in Figure 4 that a number of lower layer cognitive processes, as represented by double-ended boxes, are adopted to carry out the creation process. These supporting processes for the creation process are those of ObjectIdentification**ST**, ConceptEstablishment**ST**, Search**ST**, Qualification**ST**, and Memorization**ST** according to LRMB (Wang et al., 2006).

The Formal Model of the Creation Process

On the basis of the conceptual model as given in Figure 4, a rigorous process model of creation can be formally described, as shown in Figure 5, using RTPA (Wang, 2002b, 2007a, 2008a,

Figure 4. The cognitive process of creation

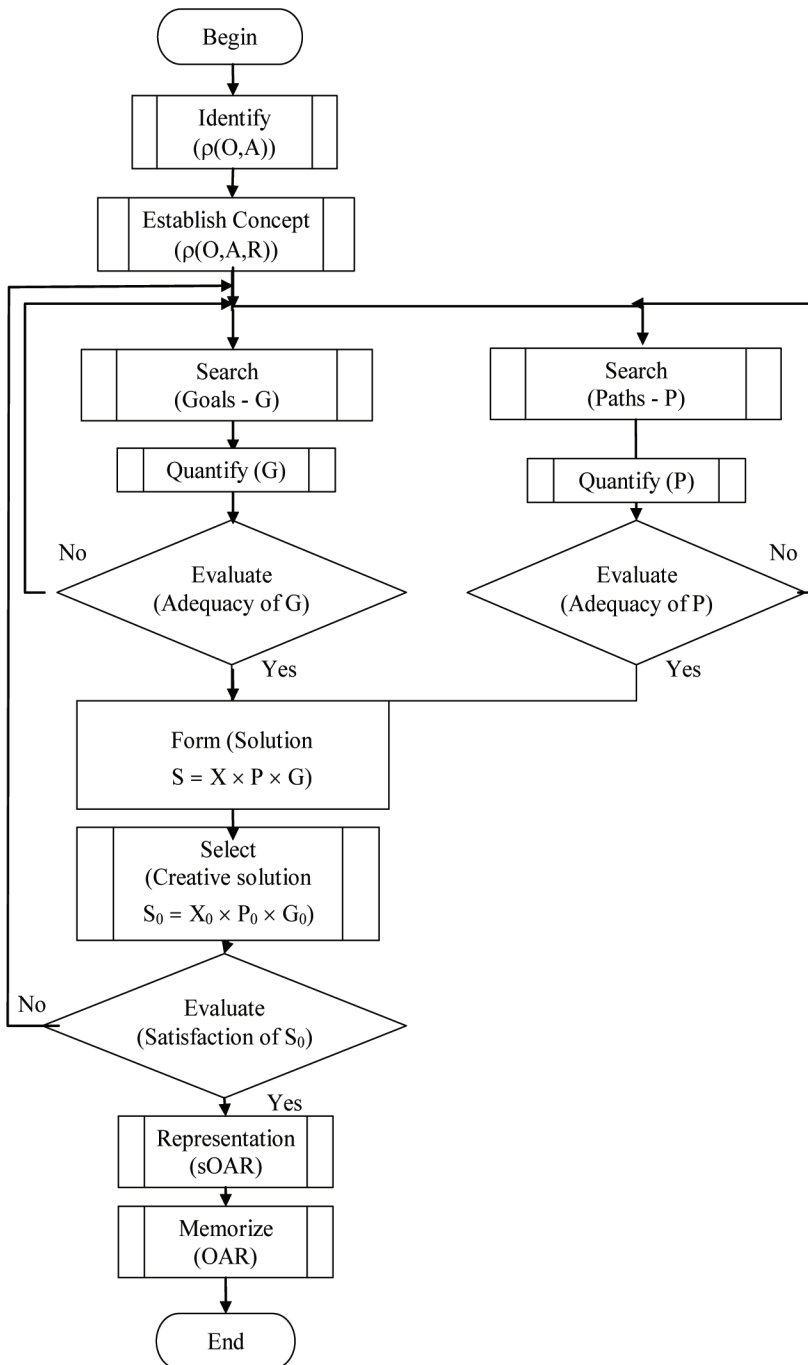
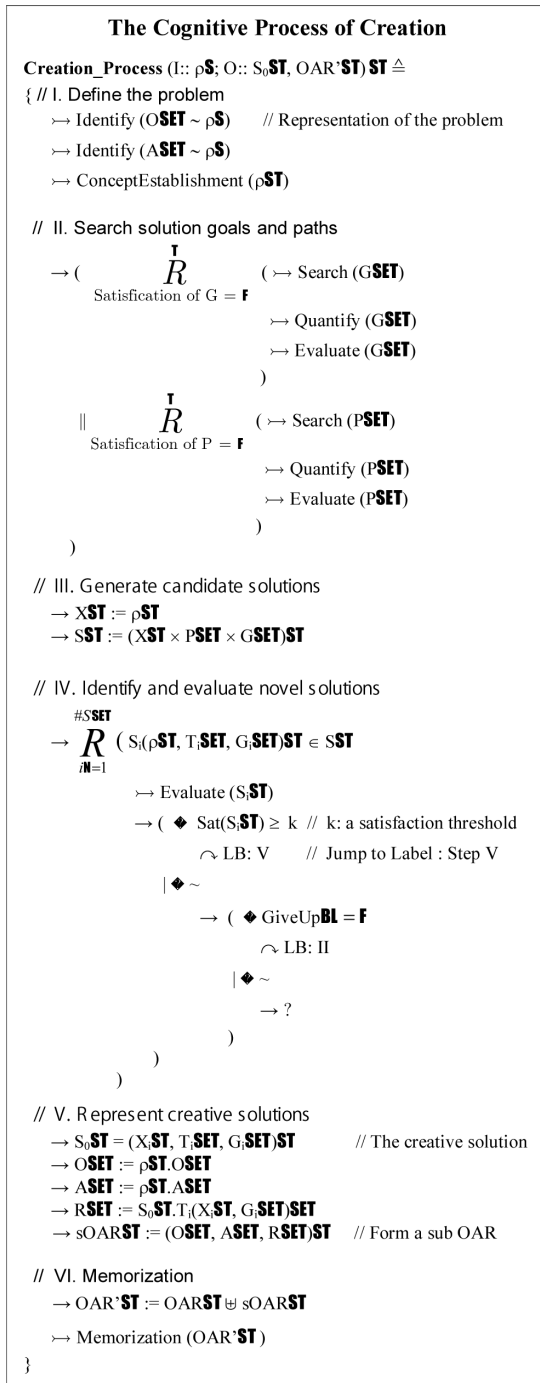


Figure 5. Formal description of the creation process in RTPA



2008e). The RTPA model formally explains the cognitive process of creation in the following six steps:

1. **To define the problem:** This step describes the problem $\rho\mathbf{S}$ by identifying the related objects \mathbf{OSET} and attributes \mathbf{ASET} . Then, a problem concept $\rho\mathbf{ST}$ in the form of a sub-OAR model $\rho(\mathbf{OSET}, \mathbf{ASET}, \mathbf{RSET})\mathbf{ST}$ is established.
2. **To search the solution goals and paths:** In this step, the brain performs a parallel search for possible goals \mathbf{GSET} and paths \mathbf{PSET} of a set of potential solutions. External memory and resources may be searched if there is no available or sufficient \mathbf{GSET} or \mathbf{PSET} in the internal knowledge of the problem solver.
3. **To generate candidate solutions:** This step forms a set of possible solutions according to Eq. 7, which is a Cartesian product of the searching results produced in Step (ii), i.e., $\mathbf{SST} = X \times T \times G, S \subseteq P$.
4. **To identify and evaluate novel solutions:** This step evaluates each potential solution in \mathbf{SST} as obtained in Step (iii) in order to find novel and creative solutions. Recursive searching actions may be executed if \mathbf{SST} cannot satisfy the originality and utility criteria for a creation.
5. **To represent creative solutions:** This step creates a new sub-OAR \mathbf{ST} to represent the creative solution(s) $S_0\mathbf{ST}, S_0\mathbf{ST} \subseteq \mathbf{SST}$, obtained in Step (iv).
6. **To memorize creative relations:** This step incorporates and memorizes the solution(s) in the form of sub-OAR \mathbf{ST} into the entire OAR \mathbf{ST} model in the long-term memory of the brain, where \uplus denotes a concept composition in long-term memory.

The cognitive process of creation developed in this section not only reveals the mechanism of basic human creation and invention process, but also indicates the approach to implement machine intelligence on creation and creative knowledge processing.

CONCLUSION

This article has presented the cognitive process of creation and creativity as a higher-level life function according to the Layered Reference Model of the Brain (LRMB). The cognitive foundations of creativity, such as the space of creativity, the approaches to creativity, the relationships of creation with problem solving, and the attributes of inventors, have been explored. A set of mathematical models of creation and creativity has been developed based on the hierarchical structures and properties of human knowledge known as concept trees. The measurement of creativity has been quantitatively analyzed. The cognitive process of creation has been described with Real-Time Process Algebra (RTPA), which provides a formal explanation of human creativity.

A creation has been defined as a novel and unexpected solution, which is a subset of the entire set of the creation space that meet the criteria of *novelty*, *originality*, and *utility*. The extent of creativity has been modeled as proportional to the product of the creative distance and the size of the creation space, subject to a satisfactory originality. Various creativities and creation processes have been identified such as free/constrained creativity, analytic/synthetic creativity, inference-based creativity, problem-solving-based creativity, and scientific/ technological/art creativity. The entire set of creativities has been classified into three categories according to their creation spaces, approaches, and problem domains.

ACKNOWLEDGMENT

This work is partially sponsored by the Natural Sciences and Engineering Research Council of Canada (NSERC). The author would like to thank the anonymous reviewers for their valuable suggestions and comments on this work.

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