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Discovering the Capacity of Human Memory

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Abstract. Despite the fact that the number of neurons in the human brain has been identified in cognitive and neural sciences, the magnitude of human memory capacity is still unknown. This paper reports the discovery of the memory capacity of the human brain, which is in the order of 10^{8432} bits. A cognitive model of the brain is created, which shows that human memory and knowledge are represented by relations, i.e., connections of synapses between neurons, rather than by the neurons themselves as the traditional container metaphor described. The determination of the magnitude of human memory capacity is not only theoretically significant in cognitive science, but also practically useful to unveil the human potential, as well as the gap between the natural and machine intelligence.

Key words: **A1**

1. Introduction

What is the memory capacity of human brains? This is a fundamental question of cognitive science, neuropsychology, and cognitive informatics. The number of neurons in an adult brain has been identified to be on the order of 100 billion (10^{11}), and each neuron is connected to a large number of other neurons via several hundreds to a few thousands synapses (Marieb, 1992; Smith, 1993; Pinel, 1997; Sternberg, 1998; Rosenzmeig *et al.*, 1999). However, the magnitude of memory capacity of human brains is still a mystery. This is mainly because the estimation of this factor is highly dependent on suitable cognitive and mathematical models of the brain.

It is commonly understood that memory is the foundation of any natural intelligence. Cognitive scientists believe that the elementary function and mechanism of the brain are quite simple; however, the magnitude of the neural networks and their concurrent behaviors are extremely powerful as a whole (Turing, 1936; Rabin and Scott, 1959; Kotulak, 1997; Leahey, 1997; Gabrieli, 1998; Matlin, 1998; Payne and Wenger, 1998; Harnish, 2002). Comparing the human brain and those of other animals, the magnitude of the human memory shows a significant difference. Therefore, to accurately determine the magnitude of human memory capacity is not only theoretically significant in cognitive science, but also practically useful to unveil the human potential. It is also helpful to perceive the status and limitations of current memory and computing technologies in computer science and artificial intelligence.

This paper explores the magnitude of human memory capacity based on a cognitive model of the brain and a set of mathematical and computational algorithms. Section 2 introduces the cognitive model of human memory. Section 3 establishes a mathematical model of memory capacity of the brain, and gives an initial estimation of the magnitude of human memory capacity. Section 4 develops a computational solution for calculating the memory capacity of the brain, which is obtained to be on the order of 10^{8432} bits. Section 5 provides additional mathematical evaluation on the estimation of the memory capacity of the brain. Section 6 explains the physical and physiological meanings of this discovery. Section 7 draws conclusions based on the finding and discusses its impact and applications in cognitive science, neuropsychology, cognitive informatics, and computing science.

2. The Cognitive Model of Human Memory

The human memory includes the sensory buffer memory, the short-term memory, the long-term memory (LTM) (Baddeley, 1990; Smith, 1993; Squire *et al.*, 1993; Sternberg, 1998; Rosenzmeig *et al.*, 1999), and the action buffer memory (Wang *et al.*, 2002; Wang and Wang, 2002). Among these memories, the LTM is the permanent memory that human beings rely on for storing acquired information such as facts, knowledge, and skills. Although cognitive science, neurophysiology, and neuropsychology have so far not been able to determine what the magnitude of the LTM capacity is, it is believed empirically that the LTM is for all intents and purposes (Smith, 1993; Rosenzmeig *et al.*, 1999; Harnish, 2002).

Model 1: The functional model of LTM can be described as a set of *hierarchical neural clusters* with partially connected *neurons* via *synapses*.

The LTM model can be illustrated as shown in Figure 1. Conventionally, LTM is perceived as static and fixed in an adult brain (James, 1890; Baddelay, 1990; Smith, 1993; Sternberg, 1998; Rosenzmeig *et al.*, 1999). This was based on the observation that the capacity of adult brains has already reached a stable state and would not grow continuously. However, latest discoveries in neuroscience and cognitive informatics indicate that LTM is dynamically reconfiguring, particularly at the lower levels of the neural clusters (Squire *et al.*, 1993; Rosenzmeig *et al.*, 1999; Wang and Wang, 2002). Otherwise we cannot explain the mechanisms of memory establishment, enhancement, and evolution that are functioning everyday in the brain.

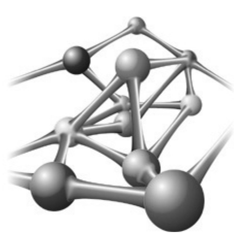

Figure 1. LTM: Hierarchical and partially connected neural clusters. The long-term memory (LTM) is dynamic and partially interconnected neural networks, where a connection between a pair of neurons by a synapse represents a *relation* between two cognitive objects.

Actually, the above perceptions are not contradictory. The former notes that the macronumber of neurons in the adult brains will not increase significantly. The latter recognizes that information and knowledge should be physically and physiologically represented in LTM by something and in some ways. Based on the latter, a cognitive model of LTM will be developed below to explain how information or knowledge is represented in LTM.

In contrast to the traditional *container* metaphor, the human memory mechanism can be described by a *relational* metaphor. The new metaphor perceives that memory and knowledge are represented by the connections between neurons in the brain, rather than the neurons themselves as information containers. Therefore, the cognitive model of human memory, particularly LTM, can be described by two fundamental artifacts:

- *Objects*: The abstraction of external entities and internal concepts. There are also subobjects known as *attributes*, which are used to denote detailed properties and characteristics of an object.
- *Relations*: Connections and relationships between object–object, object–attributes, and attribute–attribute.

Based on the above discussion, an Object–Attribute–Relation (OAR) model of memory is derived as shown below.

Model 2. The OAR model of LTM can be described as a triple, i.e.,

$$OAR \triangleq \langle o, A, R \rangle \tag{1}$$

where o is a given object identified by an abstract name, A is a set of attributes for characterizing the object, and R is a set of relations between the object and other objects or attributes of them.

An illustration of the OAR model between two objects is shown in Figure 2. It is noteworthy as in the OAR model that the *relations* themselves represent information and knowledge in the brain. The relational metaphor is totally different from the traditional container metaphor in neuropsychology and computer science, because the latter perceives that memory and knowledge are *stored* in individual neurons and the neurons function as containers.

A related technology, the semantic net, has been adopted in linguistics and artificial intelligence (Bender, 1996). The semantic net is a graphical reasoning method that adopts directed graphs to denote facts by the vertices and relationships by the edges (Bender, 1996). The OAR model may be perceived as an extended, large scale, and partially connected semantic net. However, the differences between a semantic net of knowledge and an OAR model of memory architecture are noteworthy. The former is used to denote knowledge in nonmonotonic reasoning, while the latter is designed to describe internal information and knowledge representation of the brain.

According to the OAR model, although the number of neurons in the brain is limited, the possible relations between them may result in an explosive number of combinations that represent knowledge in the human memory. Therefore, the OAR model is capable of explaining the fundamental mechanisms of human memory creation, retention, and processing.

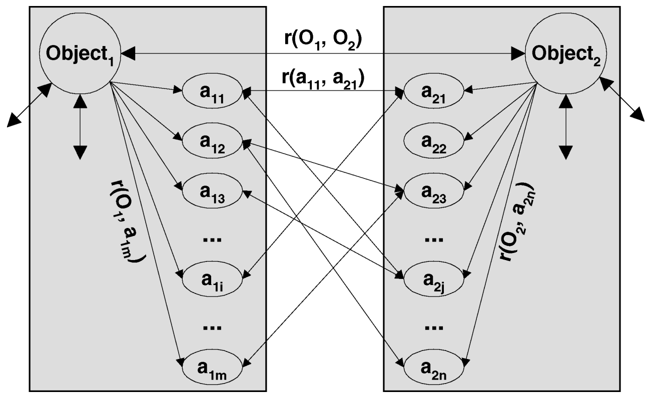

Figure 2. The OAR model of memory architecture. The relations between objects can be established via pairs of object–object, object–attribute, and/or attribute–attribute. The connections could be highly complicated, while the mechanism is so simple that it can be deducted to the physiological links of neurons via synapses in LTM.

3. The Mathematical Model of Memory Capacity of the Brain

According to the OAR model as shown in Figure 2, information is represented in the brain by relations, a conceptual model of synapses. Hence, the capacity of human memory is not only dependent on the number of neurons, but also the connections among them. This mechanism may result in an exponential combination to represent and store information in LTM of the brain. This also explains why the magnitude of neurons in an adult brain seems stable; however, huge amount of information can be remembered throughout the entire life of a person.

On the basis of the OAR model, assuming there are n neurons in the brain, and on average there are m connections between a given neuron and the rest of them, the magnitude of the brain memory capacity can be expressed by the following mathematical model, the human memory capacity model, as given below:

$$\begin{aligned} C_n^m &= \binom{n}{m} \\ &= \frac{n!}{m!(n-m)!} \end{aligned} \tag{2}$$

where n is the total number of neurons and m the number of average partial connections between neurons.

Equation (2) shows that the memory capacity problem in cognitive science and neuropsychology can be reduced to a classical combinatorial problem, with the total potential relational combinations C_n^m among all neurons ($n = 10^{11}$) and their average synapses ($m = 10^3$). Therefore, the parameters of Eq. (2) can be determined as shown in Eq. (3).

$$\begin{aligned} C_{10^{11}}^{10^3} &= \binom{10^{11}}{10^3} \\ &= \frac{10^{11}!}{10^3!(10^{11}-10^3)!} \end{aligned} \tag{3}$$

Equations (2) and (3) provide a mathematical explanation of the OAR model, which shows that the number of connections among neurons in the brain can be derived by the combination of a huge base and a large number of choices.

This seems a simple problem intuitively. However, it turns out to be extremely hard to calculate and is almost intractable using a modern computer, because of the exponential complicity or the recursive computational costs for such large n and m. However, using approximation theory, it can be estimated that the upper limit of C_n^m, when $n = 10^{11}$ and $m = 10^3$, is in the following order:

$$\begin{aligned} C_n^m &= O(n^m) \\ &\approx n^m \\ &= (10^{11})^{10^3} \\ &= 10^{11,000} \end{aligned} \tag{4}$$

Equation (4) demonstrates that the potential capacity of human memory is bounded by $10^{11,000}$ bits. This is an initial rough estimation of the magnitude of the human memory capacity.

4. A Computational Solution to the Human Memory Capacity Problem

In the previous section the mathematical model of the human memory capacity is established, and the magnitude of the capacity is estimated on the order of $10^{11,000}$. This section describes a numerical algorithm for accurately determining the capacity of human memory.

The first difficulty in solving Eq. (3) is the space complexity. To enable the large combination to be processed, a logarithm-based algorithm is developed for calculating the factorials in the memory capacity model by the following equation, i.e.,

$$\begin{aligned} \ln(n!) &= \ln\left(\prod_{i=1}^{n} i\right) \\ &= \sum_{i=1}^{n} \ln i \end{aligned} \tag{5}$$

Although the memory overflow problem can be solved by using Eq. (5), the time complexity in computing the sum of logarithms is still another obstacle to be overcome. To reduce the time complexity required to resolve Eq. (3), Eq. (2) can be rewritten as follows:

$$\begin{aligned} C_n^m &= \frac{n!}{m!(n-m)!} \\ &= \frac{\prod_{i=n-m+1}^{n} i}{m!} \end{aligned} \tag{6}$$

Transforming Eq. (6) by taking the logarithm on both sides of the expression, we obtain,

$$\ln C_n^m = \sum_{i=n-m+1}^{n} \ln i - \sum_{i=1}^{m} \ln i \tag{7}$$

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Assuming a natural logarithm and an addition operation is a unit of computation, the time complexity of Eq. (7) is reduced by $2 \times (10^{11} - 10^3)$ against that of Eq. (2). By this algorithm, a numeric solution of Eq. (6) is successfully and efficiently obtained as $C_{10^{11}}^{10^3} = 10^{8432}$, i.e.

$$\begin{aligned} C_{10^{11}}^{10^3} &= \binom{10^{11}}{10^3} \\ &= \frac{10^{11}!}{10^3!(10^{11} - 10^3)!} \\ &= 10^{8432} \end{aligned} \tag{8}$$

Comparing the above result with the upper-limit estimation given in Eq. (4), the accuracy of this result can be justified. Further evaluation of the result derived in Eq. (8) is provided in the next section.

5. A Generic and Efficient Algorithm to Estimate Huge Combinations

When m and n in the combination are getting larger than those given in Eq. (3), for example $C_{10^{11}}^{10^{10}}$, it will be very difficult to find a direct numerical solution. Therefore, an additional algorithm is developed in this section based on Trapezoidal rule (Jordan and smith, 1997), by which the solution of a huge combination can be regarded as a numerical integration problem. If a specific function can be derived to calculate the sums of Eq. (6), the computational complexity can then be simplified greatly. To create such an analytic function, linear approximation is adopted to calculate natural logarithm in the estimation, i.e.,

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}[f(a) + f(b)] \tag{9}$$

where $f(x)$ is a given function for integration, and a and b are the beginning and end of the integration interval.

The remainder of the quadrature formula, $E_t(F)$, as described in Eq. (9), is defined in the following:

$$E_t(f) = -\frac{(b-a)^3}{12} f''(\xi_t) \tag{10}$$

where $a \le \xi_t \le b$, and $f''(\xi_t)$ is the second-order derivation of $f(x)$ on ξ_t.

The linear functions in regions $\{1, 2, K, m\}$ and $\{n - m + 1, n - m + 2, K, n\}$ are given in Eqs. (11) and (12):

$$f(i) = \frac{\ln m}{m-1} \times i - \frac{\ln m}{m-1}, \quad i = 1, 2, K, m \tag{11}$$

$$f(i) = \frac{\ln n - \ln(n-m+1)}{m-1} \times i - \frac{(n-m+1)\ln n - n\ln(n-m+1)}{m-1}, \quad i = n-m+1, n-m+2, K, n \tag{12}$$

The above functions yield an approximate function as shown below:

$$\begin{aligned}\ln C_n^m &\approx \frac{m\ \ln[n(n-m+1)]}{2} - \frac{m\ \ln m}{2}\\ &= \frac{m}{2}\ \ln\frac{n(n-m+1)}{m}\end{aligned} \tag{13}$$

Using this estimation algorithm, $C_{10^{11}}^{10^3}$ is obtained as 10^{9500}. This result is quite close to the solution of Eq. (8) as derived in Section 4.

To improve the estimation of the quadrature accuracy of this algorithm, the remainder provided in Eq. (10) may be used to compensate the result obtained by Eq. (13). A simplified representation of the remainder for this specific problem can be expressed in Eq. (14), i.e.,

$$E = \left(\sum_{i=n-m+1}^{n} \ln i - \sum_{i=1}^{n} \ln i\right) - \frac{m}{2}\ \ln\frac{n(n-m+1)}{m} \tag{14}$$

Equation (14) can be transformed into the form of the following:

$$\begin{aligned}E = \sum_{i=1}^{m}\Bigg[&\ln\left(1+\frac{n-m}{i}\right)\\ &-\frac{i\ \ln n - i\ \ln(n-m+1) - i\ \ln m - \ln n + m\ \ln(n-m+1) + \ln m}{m-1}\Bigg]\end{aligned} \tag{15}$$

In Eq. (15), the ith subremainder of E_i is defined as

$$\begin{aligned}E_i = \ln&\left(1+\frac{n-m}{i}\right)\\ &-\frac{i\ \ln n - i\ \ln(n-m+1) - i\ \ln m - \ln n + m\ \ln(n-m+1) + \ln m}{m-1}\end{aligned} \tag{16}$$

where $i = 1, 2, K, m$.

It is noteworthy in Eq. (16) that the second-order derivatives of E_i, E_i'', are always positive, and E_i equals zero when i is 1 or m; it is negative when $1 < i < m$. This observation can be expressed as shown in Eqs. (17) and (18):

$$E_i'' = \frac{(n-m)(2i+n-m)}{i^2(i+n-m)} \tag{17}$$

$$E_i \begin{cases} = 0, & i = 1, m\\ < 0, & 1 < i < m\end{cases} \tag{18}$$

Thus, the sum of E_i will be always negative, which indicates that the result of the approximate algorithm derived in Eq. (13) is always greater than the accurate one.

By using linear approximation, the remainder can be estimated by the following method:

$$E \approx -\frac{1}{2}m \max_{1<i<m} |E_i| \tag{19}$$

Then, the remainder E can be used to compensate the solution to Eq. (13) in order to improve its accuracy. After considering the effect of E, an improved version of Eq. (13) is derived below:

$$\ln C_n^m \approx \frac{m}{2}\ln\frac{n(n-m+1)}{m} - \frac{1}{2}m \max_{1<i<m}|E_i| \tag{20}$$

Applying Eq. (20) to the case of $C_{10^{11}}^{10^3}$, it is obtained that $\max_{1<i<m} = 3.98$. Thus, an improved estimation of Eq. (20) after compensation is 10^{8600}, which is more closer to the result, 10^{8432}, as obtained in Section 4.

This is an indirect proof of the accuracy of the solution derived by Eq. (8). Also, this algorithm is more powerful to solve further complicated combinational problems. For example, $C_{10^{11}}^{10^{10}}$ can be efficiently calculated by this algorithm, which results in $10^{5.9\times10^{10}}$.

6. The Physical and Physiological Meaning of the Finding

The previous sections unveil that the magnitude of the memory capacity of the brain may reach an order as high as 10^{8432} bits. The magnitude of the brain capacity is believed as one of the profound advantages of human beings, which forms the quantitative foundation of natural intelligence. The other advantage of human beings is the qualitative foundation of the brain that possesses the abstract thinking layer based on the extremely large memory capacity available in the brain. The finding on the magnitude of the human memory capacity reveals an interesting mechanism of the brain. That is, the brain does not create new neurons to represent new information, instead it generates new synapses between the existing neurons in order to represent new information. The observation in neurophysiology that the number of neurons is kept stable rather than continuously increasing in adult brains (Marieb, 1992; Pinel, 1997; Rosenzmeig *et al.*, 1999) is an indirect evidence for the relational cognitive model of information representation in human memory as described in this paper.

It is interesting to contrast the memory capacities between modern computers and human beings. The capacity of computer memory (mainly the hard drives) has been increased dramatically in the last few decades from a few kilobytes to several Gigabytes (10^9 byte), even Terabytes (10^{12} byte). Therefore, with an intuitive metaphor that 1 neuron = 1 bit, optimistic venders of computers and memory chips perceived that the capacity of computer memory will, sooner or later, reach or even exceed the capacity of human memory (Sobloniere, 2002; Wang *et al.*, 2002). However, according to the finding reported in this paper, the ratio r between the brain memory capacity (C_b) and the projected computer memory capacity (C_c) in the next 10 years, is as follows:

$$\begin{aligned} r &= C_b/C_c \\ &= 10^{8432}/8\times10^{12} \\ &\approx 10^{8432}/10^{13} \\ &= 10^{8319} \end{aligned} \tag{21}$$

Equation (21) indicates that the memory capacity of a human brain is equivalent to at least 10^{8419} modern computers. In other words, the total memory capacity of computers all over the world is far more less than that of a single human brain. Equation (21) also shows the power of the OAR mechanism and configuration of the brain, which uses only 100 billion neurons and their relational combinations to represent and store up to 10^{8432} bits of information and knowledge.

The tremendous difference of memory magnitudes between human beings and computers demonstrates the efficiency of information representation, storage, and processing in the human brains. Computers store data in a direct and unconsumed manner, while the brain stores information by relational neural clusters. The former can be accessed directly by explicit addresses and can be sorted, while the latter may only be retrieved by content-sensitive search and matching among neuron clusters where spatial connections and configurations themselves represent information.

7. Conclusions

Investigation into the memory capacity of the brain has been perceived to be one of the fundamental research areas that help to unveil the mechanisms and the potential of the brain, and to provide a reference model of information representation and storage for computing and information sciences and the information technology industry. This paper has explored the magnitude of human memory capacity based on the OAR model and a set of mathematical and computational algorithms. The computational solution to the memory capacity of the brain has been obtained as on the order of 10^{8432} bits, which is a magnitude that is very much higher than the total memory capacity of all computers ever available in the world. The discovery of this paper has demonstrated that the magnitude of human memory capacity is excessively higher than those of computers on an order that we never realized. This new factor has revealed the tremendous quantitative gap between the natural and machine intelligence. The finding of this paper has also indicated that the next generation computer memory systems may be built according to the relational (OAR) model rather than the traditional container metaphor, because the former is more powerful, flexible, and efficient, and is capable of generating a mathematically unlimited memory capacity by using limited number of neurons in the brain or hardware cells in the next generation computers.

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