

Subspace Identification Method for Ankle Mechanics

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Abstract— This paper presents results a new algorithm for modeling biological systems using a state space approach. One of the MIMO Output-Error State Space identification (MOESP) [5] family of algorithms has been adapted to model the linear part of human ankle mechanics. A mixed causal/anti-causal state space model was calculated to examine the relationship between torque and position of the ankle. Simulated and real data are used to show that the method yields parametric models for ankle mechanics that describe behavior as well as nonparametric models. However, the parametric models obtained using the MOESP identification method have many fewer parameters.

I. INTRODUCTION

Nonparametric identification methods provide convenient, robust means of characterizing the dynamics of linear systems without requiring *a priori* assumptions regarding the system structure. Our laboratory has made extensive use of the Impulse Response Function (IRF) to study ankle dynamics [2]. However, nonparametric estimates of dynamics are difficult to relate to the structure and parameters of the underlying physiological system. Parametric methods (e.g., ARMA, OE, BJ) yield results which are directly related to system structure but generally require *a priori* assumptions about the system order. This is clearly undesirable in biological applications where determining the system order is often one of the objectives. For example, in the analysis of joint mechanics, the *a priori* assumption of a second order model, which is frequently made, precludes the possibility of detecting a reflex contribution. What is needed, therefore, is a parametric method for system identification which provides an estimate of both the system order and the dynamics. The MOESP family of subspace identification algorithms shows great promise in this respect. This is a robust state space algorithm that calculates a parametric model and the system order simultaneously. Moreover, MOESP is capable of calculating a mixed causal/anti-causal model [4]. In this paper we describe the application of a variant of this method to a biological application of interest in our laboratory — the identification of human ankle dynamics.

The goal of the exercise is to characterize ankle mechanics in terms of the dynamic relation between position and torque. In principal this may be done either in terms of stiffness, taking position as input and torque as the output, or as compliance where torque is taken as the input and position as the output. In practice, it is

highly desirable to use position as the experimental input since joint torque records are likely to contain substantial noise components due to fluctuations in voluntary drive to the muscles. Nevertheless, it is often necessary to convert between the two formulations for analysis purposes. The objective of the work presented here was to develop parametric methods for estimating joint stiffness and converting it to compliance.

II. METHODS

One practical difficulty with the stiffness formulation is that the resulting system dynamics are high-pass and the associated IRF is non-causal. We have used the method shown schematically in Figure 1 to deal with this problem.

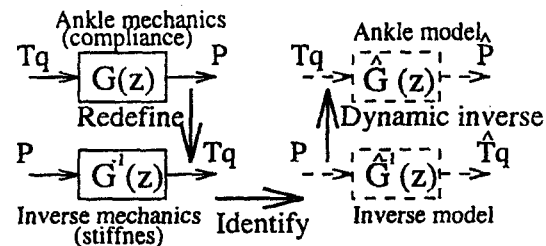


Fig. 1. Schematic of used identification method.

- Joint mechanics were defined as stiffness, to have position input and torque output.
- The stiffness model was calculated using the mixed causal/anti-causal MOESP algorithm [4] which estimates the parameters of the state space description.

$$\begin{aligned} x_{k+1}^c &= Ax_k^c + B^c P_k \\ x_{k-1}^{ac} &= Ex_k^{ac} + B^{ac} P_k \\ \hat{T}q_k &= C^c x_k^c + C^{ac} x_k^{ac} + D P_k \end{aligned}$$

where the superscripts *c* and *ac* stand for *causal* and *anticausal* respectively, *P* is the *input position*, *x* is the *state* and $\hat{T}q$ the *estimated output torque*.

- The dynamic inverse of this model was calculated analytically to obtain the compliance model.
- As a measure of model accuracy, the Variance Accounted For (VAF) of the estimated output was calculated. $\%VAF = [1 - \frac{\text{var}(P - \hat{P})}{\text{var}(P)}] 100\%$.

This indirect route to calculate the system matrices bypasses the bias problems associated with input noise [3], by taking the position as the input signal. The data was cross validated to ensure that no noise was fitted.

III. RESULTS

This new method for system identification of ankle mechanics was validated using simulated and real data with various sampling rates and Signal to Noise Ratios (SNR).

A. Simulated Case

Initially the method was evaluated with simulated data from a second order system [1] with the transfer function $\frac{1}{Is^2 + Bs + K}$ with values for I (inertia) = 0.1, B (viscosity) = 1 and K (elasticity) = 100. The model was simulated in SimulinkTM using the Runga-Kutta5 integration method and a 1kHz sampling rate. This noise free compliance model was simulated, uncorrelated zero mean Gaussian noise to the torque, and the identification carried out taking position as its input and its noise corrupted torque as the output. Figure 2 shows that even with 10dB SNR

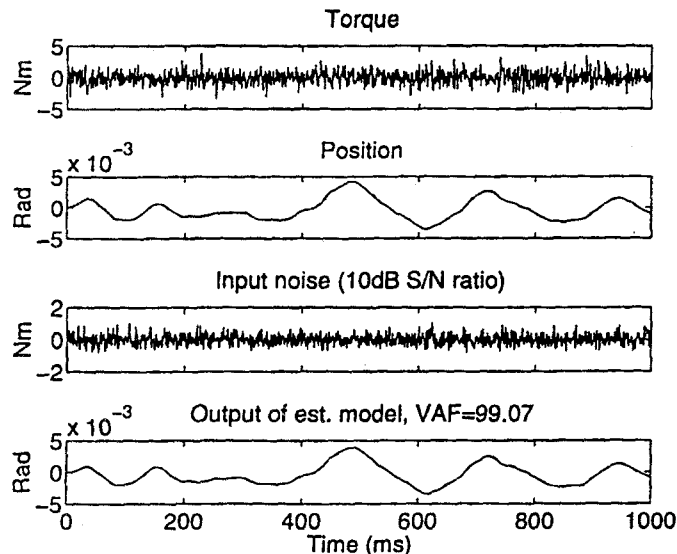


Fig. 2. Simulated case with 10dB noise.

the model prediction was still very accurate (99% VAF). Thus, the method appears to be a robust scheme for system identification in the presence of output noise.

B. Human Ankle Mechanics

The MOESP [3] algorithm was then tested on experimental data from our laboratory. Figure 3 shows the output data and fits obtained using the mixed MOESP and the classical IRF methods. The model fit obtained using a third order mixed causal/anti-causal MOESP method having 7 free parameters was as good as that obtained with the IRF with 70 lags; VAF=96.52% versus 96.24% (see Fig. 3). In addition, the new method provides a direct way to estimate the system parameters, which was not possible with the nonparametric method. However, it is yet not possible to convert these parameters to their continuous time equivalents.

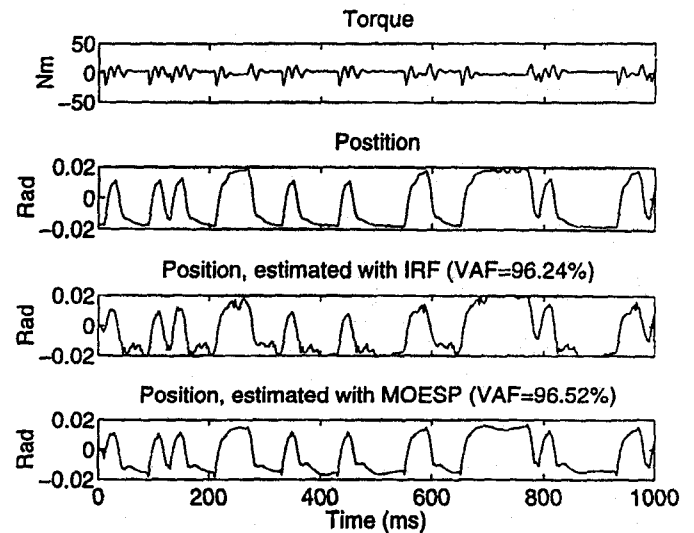


Fig. 3. Real case for the forward system.

IV. DISCUSSION

The results from these case studies demonstrate that our new mixed causal/anti-causal identification method has great promise. The MOESP identification method is a robust method for biological system modeling, resistant to noise problems presented by real data and suitable for non-causal dynamics. Further analysis of this identification scheme is necessary to develop means to convert the discrete time parameters to their continuous time equivalent. Moreover, it will be necessary to study the effects of different types of data on this algorithm to understand fully its behavior. Further studies will examine more experimental data for a range of subjects with different SNR's, varying sampling rates and various input bandwidths to assess the robustness and consistency of the algorithm.

V. ACKNOWLEDGMENTS

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REFERENCES

- [1] I.W. Hunter and R.E. Kearney. Dynamics of human ankle stiffness: Variation with mean ankle torque. *J. Biomechanics*, 15(10):747-752, 1982.
- [2] I.W. Hunter and R.E. Kearney. Two-sided linear filter identification. *Med. & Biol. Eng. & Comput.*, 21:203-209, 1983.
- [3] Michel Verhaegen. Subspace model identification, part3. analysis of the ordinary output-error state-space model identification algorithm. *Int. J. Control*, 58:555-586, 1993.
- [4] Michel Verhaegen. A subspace model identification solution to the identification of mixed causal, anti-causal lti systems. *SIAM J. matrix analysis and applications*, 1994.
- [5] Michel Verhaegen and Patrick Dewilde. Subspace model identification part 1. the output-error state-space model identification class of algorithms. *Int. J. Control*, 56(5):1187-1210, 1992.